**23.** A vessel is to be loaded with stocks of 3 items. Each unit of item $i$ has a weight $w_i$ and value $r_i$. The maximum cargo weight the vessel can take is 5 and the details of the three items is as follows :

| $i$ | $w_i$ | $r_i$ |
|---|---|---|
| 1 | 1 | 30 |
| 2 | 3 | 80 |
| 3 | 2 | 65 |

Develop the recursive equation for the above case and find the most valuable cargo load without exceeding the maximum cargo weight by using dynamic programming.
[**Ans.** Optimal solution : (1, 0, 2); total value = 160]

**24.** A firm of manufacturers stock up every two months with certain basic material in order to carry out its production schedule. The purchase price $p_n$ and the demand $d_n$, $n = 1, 2, \ldots, 6$ are given for the next bi-monthly periods the stock must never exceed a certain value S. The initial stock is 2 and the final stock must be nil. Use dynamic programming to ascertain the quantities to be bought at the beginning of each period in such a way that the total cost will be minimum.

| Period ($n$) : | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Demand ($d_n$) : | 8 | 5 | 3 | 2 | 7 | 4 |
| Purchase price ($p_n$) : | 11 | 18 | 13 | 17 | 20 | 10 |

[**Ans.** $x_1 = 7$, $x_2 = 4$, $x_3 = 9$, $x_4 = 3$, $x_5 = 0$, $x_6 = 4$; and the minimum purchase cost is 357, where $x_n$ is the quantity bought at the beginning of $n$th period.]

**25.** A Chairman of a certain political party is making plans for his election to the parliament. He has received the services of six volunteer workers and wishes to assign them four districts in snch a way as to maximize their effectiveness. He feels that it would be inefficient to assign a worker to more than one district but he is willing to assign no worker to any one of the districts, if they can accomptish more in other districts.

Use dynamic programming technique to determine how many of the six workers should be assigned to each of the four districts in order to maximize the total estimated increase in the number of votes in his favour.

| | District | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 25 | 20 | 33 | 13 |
| 2 | 42 | 38 | 43 | 24 |
| 3 | 55 | 54 | 47 | 32 |
| 4 | 63 | 65 | 50 | 39 |
| 5 | 69 | 73 | 52 | 45 |
| 6 | 74 | 80 | 53 | 50 |

**26.** Show how the functional equations technique of dynamic programming can be used to solve the travelling sales man problem.

Hence solve the travelling salesman problem whose distance matrix is :

| | | To City | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| From City | A | — | 4 | 7 | 3 | 4 |
| | B | 4 | — | 6 | 3 | 4 |
| | C | 7 | 6 | — | 7 | 5 |
| | D | 3 | 3 | 7 | — | 7 |
| | E | 4 | 4 | 5 | 7 | — |

and city $A$ is the home city.

**27.** An equipment-rental business is considering the investment of an initial capital of Rs. 10,000 in buying two types of equipment. If $x$ is the amount of money invested in type I, the corresponding profit at the end of the first year is $g_1(x) = 5x$ while the profit from type II is $g_2(10{,}000 - x) = 7(10{,}000 - x)$. The company's policy is to salvage equipment after one year. The salvage values for type I and type II at period $t$ are $p_i x$ and $q_i(10{,}000 - x)$, where $\alpha\, p_i < 1$ and $0 < q_i < 0$. At the end of each year, the company re-invests the returns from salvaging the equipment. This is repeated over the next 4 years with the same return functions $g_1$ and $g_2$ holding for every year. Using dynamic programming, determine how much money should be invested in each equipment for the next four years, given the following data :

| $t$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p_i$ | .5 | .9 | .4 | .5 |
| $q_i$ | .6 | .1 | .5 | .7 |

**28.** A 4 ton vessel is loaded with one or more of three items. The following table gives the unit weight, $w_i$ in tons and unit revenue $r_i$ in thousands of rupees for item $i$. How should the vessel be loaded to maximize the total return ? Use dynamic programming approach.

| Item $i$ | $w_i$ | $r_i$ |
|---|---|---|
| 1 | 4 | 70 |
| 2 | 1 | 20 |
| 3 | 2 | 40 |

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